Computational Homogenization of Flows in Porous Media With Memory

Petr N. Vabishchevich

Lomonosov Moscow State University, Moscow

vab@cs.msu,ru

This study introduces a computational homogenization technique for modeling unsteady flows of an incompressible viscous fluid through periodic porous media with memory effects. On the macroscopic level, the flow is described by an integro-differential Darcy law whose tensorial memory kernel is obtained from unsteady cell problems. Our method determines the steady-state and transient components of the conductivity tensor by solving auxiliary boundary-value and spectral problems on the representative periodic cell. A key numerical step involves approximating the memory kernel with a Prony series (sum of exponentials), which reduces the nonlocal macroscopic problem to a local system of differential equations. We outline the spatial finite-element discretization and develop stable two-level time-stepping schemes. Finally, we demonstrate the proposed framework on a two-dimensional test problem, showcasing its effectiveness for unsteady filtration applications.

Keywords: unsteady flows in porous media, memory Darcy law, integro-differential equation, exponential sum approximation, two-level time discretization schemes